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MEMORANDUM REPORT NO. 1462

MARCH 1963

A METHOD FOR COMPUTING THE INTERACTION OF TWO SPHERICAL BLAST WAVES

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RDT & E Project No. 1M010501A006

A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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RCMakino/jdk
Aberdeen Proving Ground, Md.
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ABSTRACT

The artificial viscosity method of von Neumann and Richtmyer for numerical solution of the partial differential equations of fluid flow is used to obtain a scheme for the numerical calculation of the interaction of two spherical blast waves in air.

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LIST OF SYMBOLS

SUPERSCRIPTS

n refers to the time index.

o refers to time zero.

SUBSCRIPTS

k refers to the R space index.

ℓ refers to the Z space index.

a refers to charge a.

b refers to charge b.

GREEK

α = an adjustable constant to steepen shock fronts.

ρ = density.

ROMAN

c = sound velocity.

E = specific internal energy.

$$J = \begin{vmatrix} \frac{\partial r}{\partial R} & \frac{\partial r}{\partial Z} \\ \frac{\partial z}{\partial R} & \frac{\partial z}{\partial Z} \end{vmatrix} .$$

$$K = \begin{vmatrix} \frac{\partial P}{\partial R} & \frac{\partial P}{\partial Z} \\ \frac{\partial z}{\partial R} & \frac{\partial z}{\partial Z} \end{vmatrix} .$$

$$L = \begin{vmatrix} \frac{\partial P}{\partial R} & \frac{\partial P}{\partial Z} \\ \frac{\partial r}{\partial R} & \frac{\partial r}{\partial Z} \end{vmatrix} .$$

p = pressure.

P = p + q.

$$q = \begin{cases} \dfrac{\alpha\,(\rho^o)^2\,J}{\rho^3}\left(\dfrac{\partial\rho}{\partial t}\right)^2 & \text{if} \quad \dfrac{\partial\rho}{\partial t} > 0\,, \\ 0 \quad \text{if} \quad \dfrac{\partial\rho}{\partial t} \leq 0\,. \end{cases}$$

r = Eulerian coordinate perpendicular to the z-axis of symmetry.

r_a^o = initial radius of charge a.

r_b^o = initial radius of charge b.

R = Lagrange coordinate perpendicular to the Z-axis of symmetry.

ΔR = increment of Lagrange coordinate R.

t = time.

Δt = increment of time.

$u = \dfrac{\partial r(t,R,Z)}{\partial t}$ = r component of velocity.

$v = \dfrac{\partial z(t,R,Z)}{\partial t}$ = z component of velocity.

$w^o = \rho^o\, r^o\, J^o$.

z = Eulerian coordinate of cylindrical symmetry.

z_a^o = initial axial distance of center of charge a.

z_b^o = initial axial distance of center of charge b.

Z = Lagrange coordinate of cylindrical symmetry.

ΔZ = increment of Lagrange coordinate Z.

INTRODUCTION

Axi-symmetric unsteady compressible flow problems arise extensively in military technology. Spherical blast waves and their interaction, explosions in vertically variable atmosphere, flow past axi-symmetric bodies, high-speed flow in axi-symmetric hypervelocity guns, shaped-charge gas-metal jet flow--all are of importance. The exact solution of the equations governing these flows which satisfies the appropriate initial and boundary conditions requires solving a system of non-linear hyperbolic partial differential equations in a two-space and one-time coordinate system, i.e., in three generalized dimensions. Presently, such exact solutions are not obtainable. The method of characteristics by which existence of solutions is shown opens the way for a numerical procedure, but this method requires a series of machine programs that escalates in complexity as the shock and characteristic discontinuity surfaces in the flow field interact and multiply the number of continuous zones bounded by discontinuity surfaces.

Von Neumann and Richtmyer proposed a method of numerical integration that changes the flow equations to the parabolic type. Although the solution is only approximate, the overall or essential nature of the solution is preserved; i.e., curves with momentum and energy discontinuities are warped into smooth but rapidly changing curves. (Stream surface material discontinuities, however, are not altered.) Therefore, the numerical procedure does not require the separate treatment of every continuous zone, and numerical solution with today's computing machines is quite feasible.

This is the method to be examined for calculation of the interaction of the blast waves from two spherical charges of initial radii r_a^o and r_b^o, since in this interaction the flow field is replete with discontinuity surfaces that complicate the problem.

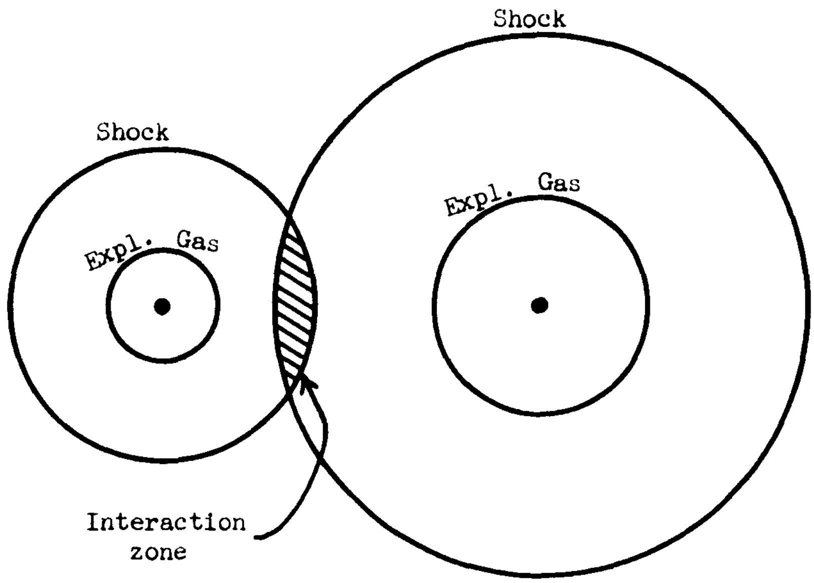

FIGURE 1. BLAST INTERACTION

While this work is the theoretical extension of the experimental work done in References 1, 2, and 3 on project CLAW, with small modifications, the analysis can be extended to the other military problems previously listed.

FLOW EQUATIONS

The continuity equation in particle or Lagrange cylindrical coordinates for axi-symmetric flow is

$$(1) \qquad \frac{\partial}{\partial t}(\rho r J) = 0, \quad J = \begin{vmatrix} \frac{\partial r}{\partial R} & \frac{\partial r}{\partial Z} \\ \frac{\partial z}{\partial R} & \frac{\partial z}{\partial Z} \end{vmatrix} ,$$

or

$$(2) \qquad \rho r J = \rho^o \, r^o \, J^o = w^o .$$

The momentum conservation equations are

$$(3) \qquad \begin{cases} \dfrac{\partial u}{\partial t} = - \dfrac{r}{w^o} K , \quad K = \begin{vmatrix} \frac{\partial P}{\partial R} & \frac{\partial P}{\partial Z} \\ \frac{\partial z}{\partial R} & \frac{\partial z}{\partial Z} \end{vmatrix} , \\[2ex] \dfrac{\partial v}{\partial t} = \dfrac{r}{w^o} L , \quad L = \begin{vmatrix} \frac{\partial P}{\partial R} & \frac{\partial P}{\partial Z} \\ \frac{\partial r}{\partial R} & \frac{\partial r}{\partial Z} \end{vmatrix} , \end{cases}$$

where

$$(4) \qquad \begin{cases} \dfrac{\partial r}{\partial t} = u , \\[2ex] \dfrac{\partial z}{\partial t} = v . \end{cases}$$

The energy conservation equation is

$$(5) \qquad \frac{\partial E}{\partial t} = \frac{P}{\rho^2} \frac{\partial \rho}{\partial t} ,$$

where, in accordance with Reference 4, P is defined by

$$(6) \qquad P = p + q \ ,$$

and q represents some artificial dissipative term. In accordance with References 5 and 7 we put

$$(7) \qquad q = \begin{cases} \dfrac{\alpha\,(\rho^{o})^{2}\,J}{\rho^{3}}\left(\dfrac{\partial\rho}{\partial t}\right)^{2} & \text{if} \quad \dfrac{\partial\rho}{\partial t} > 0 \ , \\[2ex] 0 \quad \text{if} \quad \dfrac{\partial\rho}{\partial t} \le 0 \ . \end{cases}$$

The conservation equations above are supplemented by the thermodynamic equations of state

$$(8) \qquad \begin{aligned} p &= p(E,\rho) \ , \\ c &= c(E,\rho) \ , \end{aligned}$$

which are assumed to be known.

The conservation equations above are hydrodynamically exact for $q = 0$; but for $q \neq 0$, the momentum and energy equations are modified, while the mass equation remains intact.

Equations (2) thru (8) are ten equations with the ten unknowns $(r, z, u, v, E, P, p, q, \rho, c)$, which are to be determined as functions of (t, R, Z).

INITIAL AND BOUNDARY CONDITIONS

Two explosive charges of radii r_a^o and r_b^o with centers at z_a^o and $z_b^o = -z_a^o$ along the Z-axis (Figures 2 and 3), are initiated simultaneously at their centers. Assuming equilibrium flow and steady state Chapman-Jouget propagation of the detonation front, the initial explosion state can be calculated either

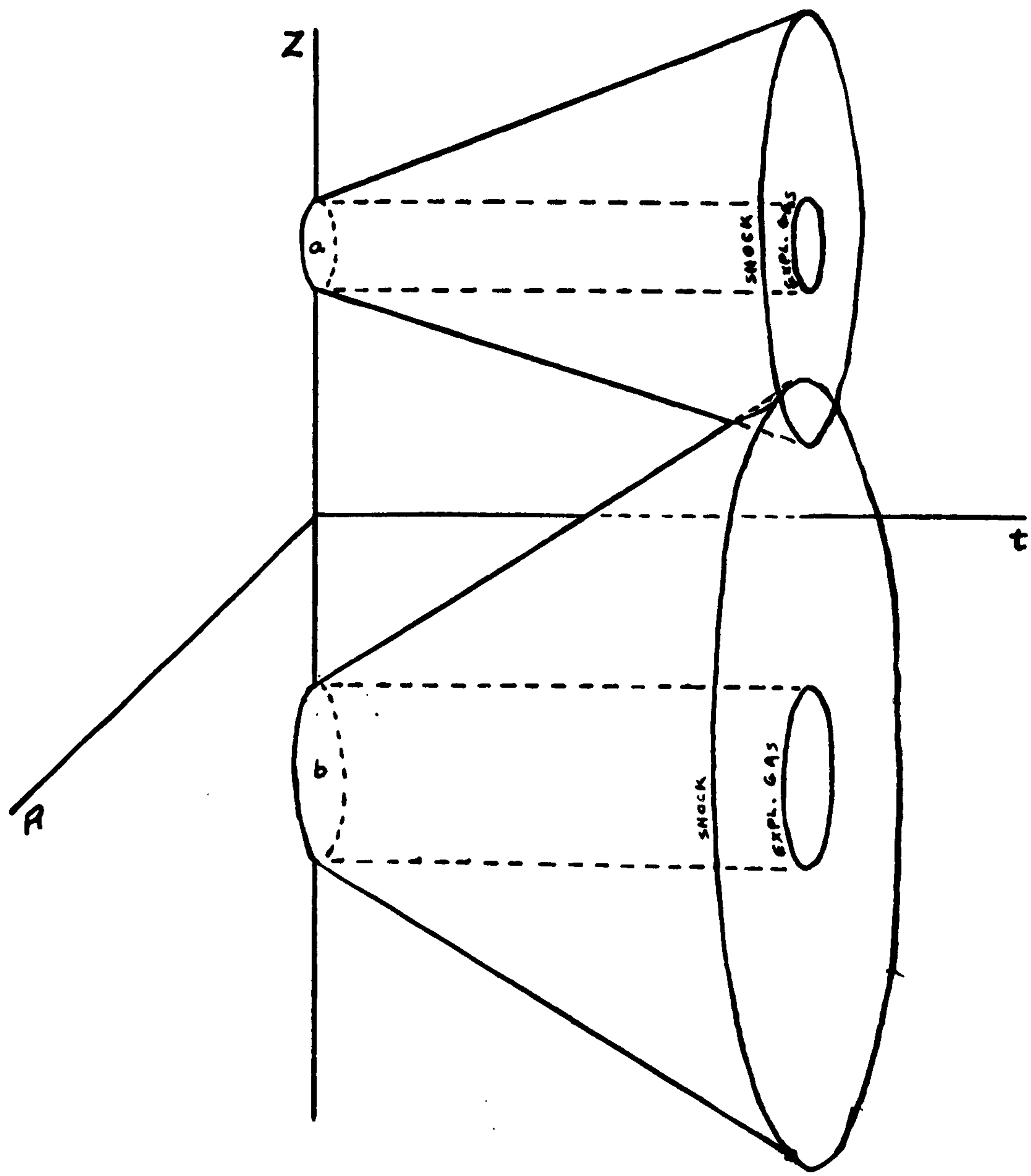

FIGURE 2. LAGRANGE SPACE

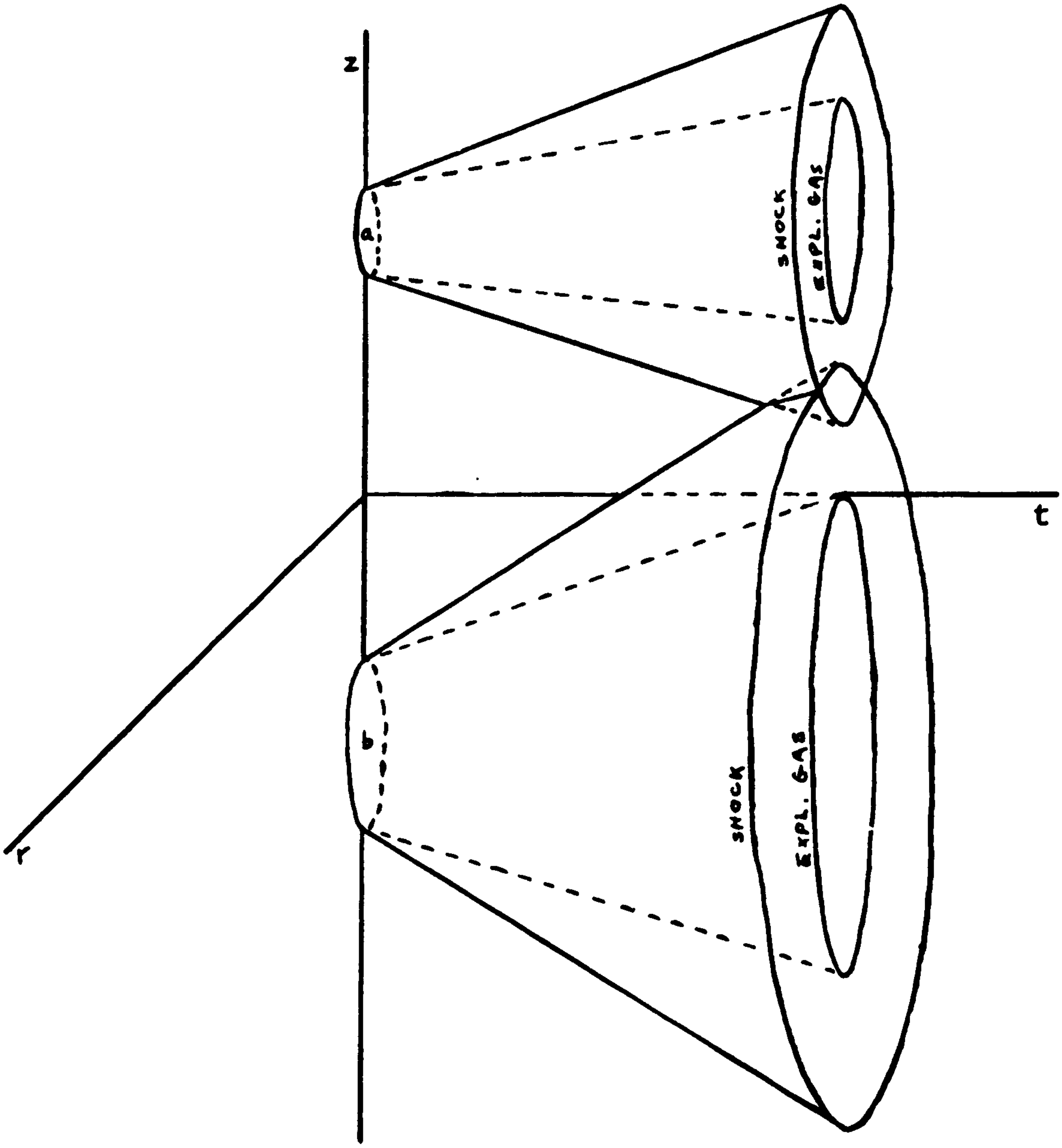

FIGURE 3. EULERIAN SPACE

by the Taylor progressive-wave approximation (Ref. 8), or by the numerical scheme to be subsequently described. In either case, the equation of state of the explosion products in chemical and thermodynamic equilibrium must be specified.

If the detonation process is to be calculated by the method outlined in this article, considering that the media ahead and behind the detonation front are not the same, the wave fronts must be treated as boundaries moving with the constant Chapman-Jouget velocity. The wave fronts form cones in the R, Z, t space (see Figure 2). Behind the detonation fronts the calculations use the equation of state of the explosion products. Ahead of the fronts are the undetonated portions of the charges.

When the detonation process is complete and the two explosion gases expand into the surrounding ambient air, shock waves are formed and propagate in an outward direction. These shocks eventually collide, and the two flows interact. The primary area of interest is in the nature of the interaction.

On each t = constant plane (Figure 4) after the shock formation, two types of domains must be considered -- the explosion gas and air domains. Each has its own equations of state (8). Although shock surfaces are moving boundaries of discontinuities, the momentum and energy equations modified by the higher derivative dissipative term no longer permit this type of discontinuity surfaces to exist. In essence, such boundaries are smoothed out and no longer distinct.

In the Lagrange space (Figures 2 and 4), each explosion gas boundary remains a fixed circle in each t = constant plane. The equations of these circles are

$$(9) \quad \begin{cases} R^2 + (Z - z_a)^2 = (r_a)^2 , \\ R^2 + (Z - z_b)^2 = (r_b)^2 . \end{cases}$$

Across these circles we assume dynamic equilibrium and laminar flow so that pressures and velocities normal to the stream surfaces are continuous. The stream surfaces remain as characteristics across which thermodynamic variables

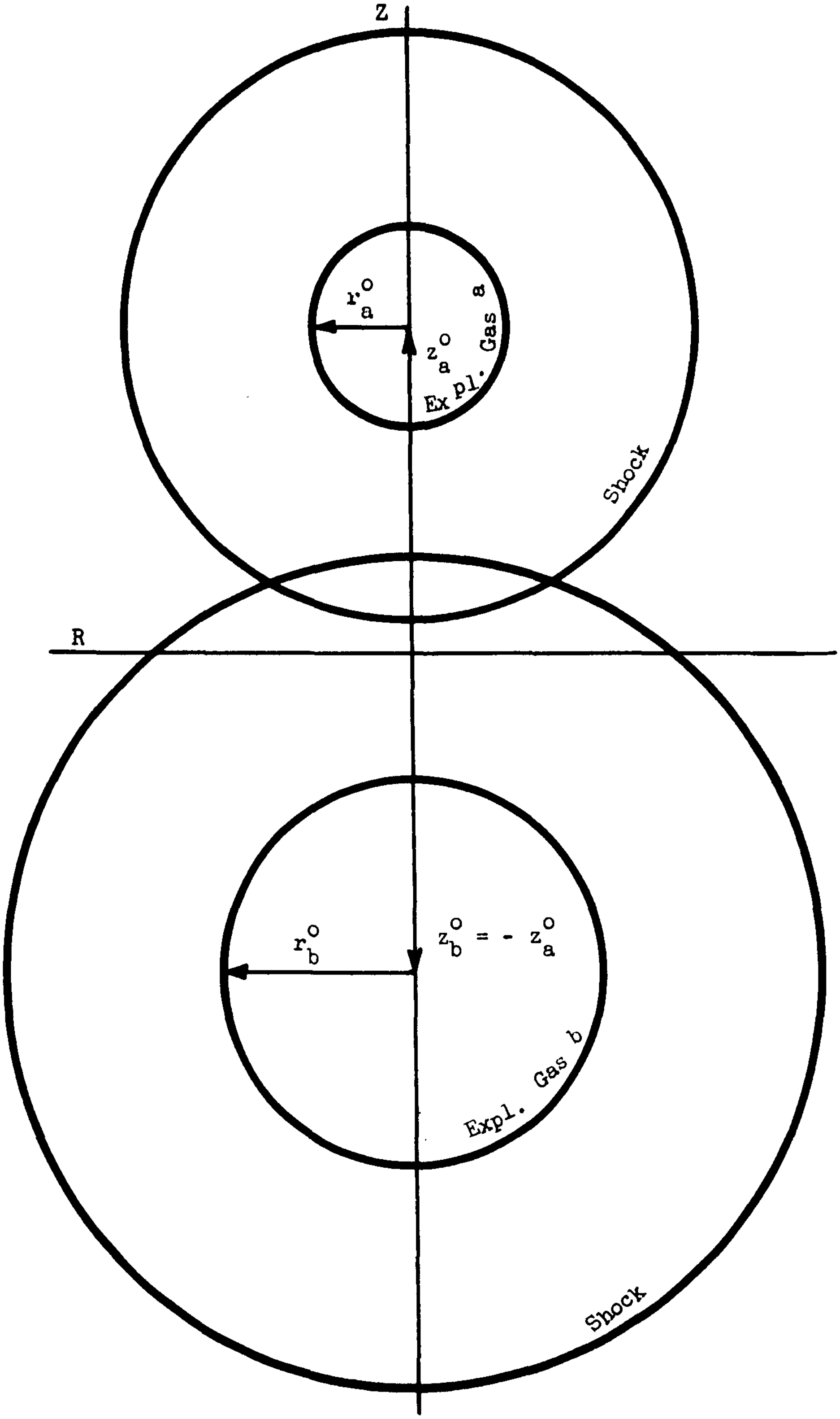

FIGURE 4. t = CONSTANT PLANE

may be discontinuous, because the mass equation is unaltered in the modified system of conservation equations. Thus, the chemical composition and equations of state change abruptly across the explosion gas-air boundary.

COMPUTATIONAL SCHEME

The computational scheme for points interior to boundaries, i.e., points other than explosion gas-air boundaries, will be that adapted to this particular problem from Reference 5. The stability of this scheme is shown in Reference 6.

Let k, ℓ, n be indices corresponding to the coordinates R, Z, t (Figure 5). Assume that on the t= 0 or n = 0'th plane all quantities, including

$$
(10)\quad \left\{
\begin{aligned}
& w^{o}_{k-\frac{1}{2},\ \ell-\frac{1}{2}} = \rho_{k-\frac{1}{2},\ \ell-\frac{1}{2}}\ r^{o}_{k-\frac{1}{2},\ \ell-\frac{1}{2}}\ J^{o}_{k-\frac{1}{2},\ \ell-\frac{1}{2}} \ , \\
& w^{o}_{k,\ell} = \frac{1}{4}\left(w^{o}_{k-\frac{1}{2},\ \ell-\frac{1}{2}} + w^{o}_{k-\frac{1}{2},\ \ell+\frac{1}{2}} + w^{o}_{k+\frac{1}{2},\ \ell-\frac{1}{2}} + w^{o}_{k+\frac{1}{2},\ \ell+\frac{1}{2}}\right) \ , \\
& \Delta R_{k,\ell} = \frac{1}{4}\left(\Delta R_{k-\frac{1}{2},\ \ell-\frac{1}{2}} + \Delta R_{k-\frac{1}{2},\ \ell+\frac{1}{2}} + \Delta R_{k+\frac{1}{2},\ \ell-\frac{1}{2}} + \Delta R_{k+\frac{1}{2},\ \ell+\frac{1}{2}}\right) \ , \\
& \Delta Z_{k,\ell} = \frac{1}{4}\left(\Delta Z_{k-\frac{1}{2},\ \ell-\frac{1}{2}} + \Delta Z_{k-\frac{1}{2},\ \ell+\frac{1}{2}} + \Delta Z_{k+\frac{1}{2},\ \ell-\frac{1}{2}} + \Delta Z_{k+\frac{1}{2},\ \ell+\frac{1}{2}}\right) \ ,
\end{aligned}
\right.
$$

are known at the pre-determined points

$$
R_{k-\frac{1}{2},\ \ell-\frac{1}{2}},\ Z_{k-\frac{1}{2},\ \ell-\frac{1}{2}} \quad \text{and} \quad R_{k,\ \ell},\ Z_{k,\ell} \ .
$$

(This grid spacing in the R, Z space may require changing at some time t^n, depending on the gradients of the solution.) Also, assume that on the successive $(n-\frac{1}{2})$th and (n-1)th constant time planes, the following quantities have been computed for all values of the indices k, ℓ up to k, ℓ:

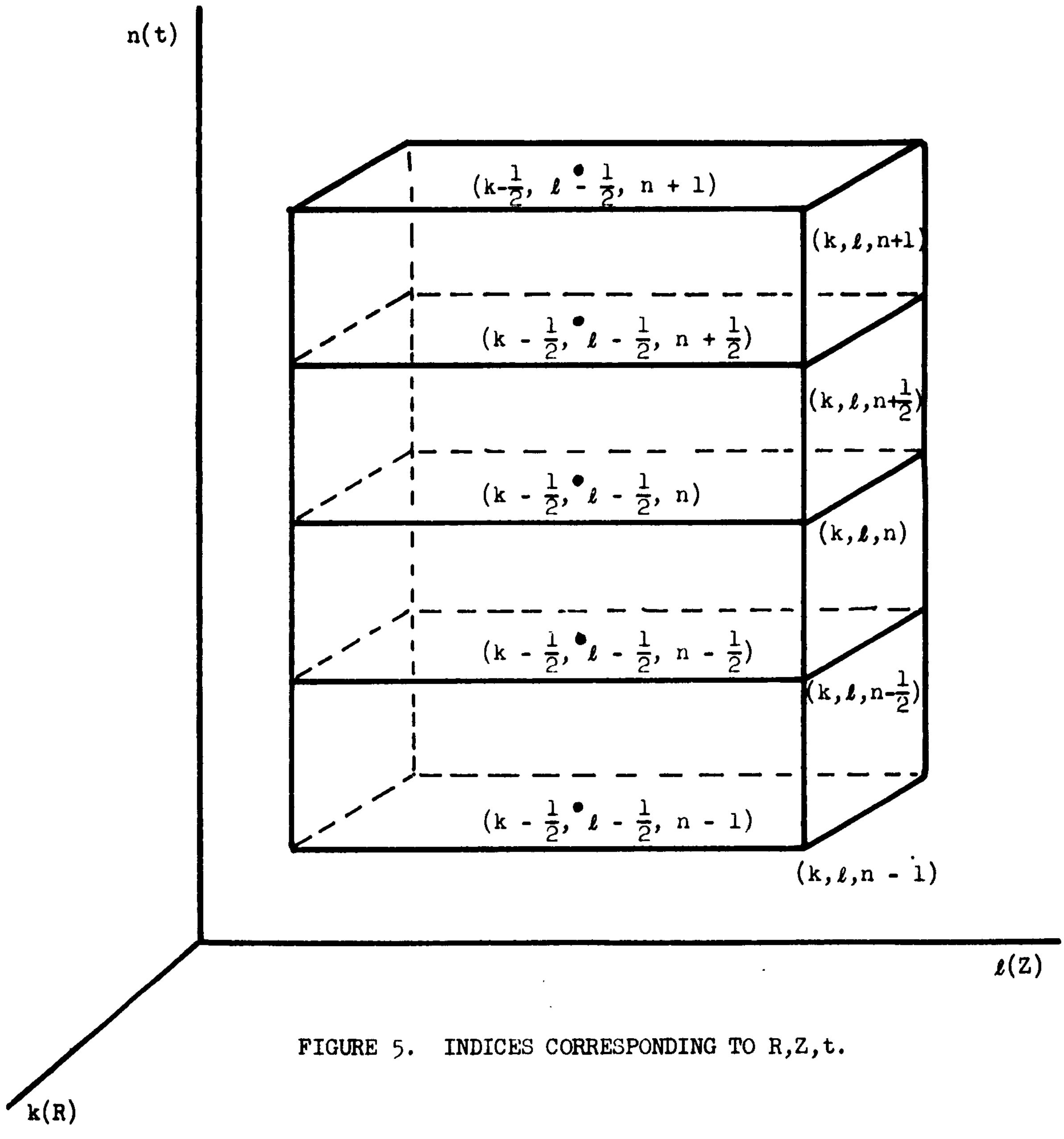

FIGURE 5. INDICES CORRESPONDING TO R,Z,t.

$(n-\frac{1}{2})$th plane

$$(\Delta t)^{n-\frac{1}{2}} \quad ,$$

$$u^{n-\frac{1}{2}}_{k-\frac{1}{2},\, \ell-\frac{1}{2}} \;,\; v^{n-\frac{1}{2}}_{k-\frac{1}{2},\, \ell-\frac{1}{2}} \; .$$

$(n-1)$th plane

$$t^{n-1} \quad ,$$

$$r^{n-1}_{k,\ell} \;,\; z^{n-1}_{k,\ell} \;,$$

$$E^{n-1}_{k-\frac{1}{2},\, \ell-\frac{1}{2}} \;,\; \rho^{n-1}_{k-\frac{1}{2},\, \ell-\frac{1}{2}} \;,\; P^{n-1}_{k-\frac{1}{2},\, \ell-\frac{1}{2}} \;,\; p^{n-1}_{k-\frac{1}{2},\, \ell-\frac{1}{2}} \;,\; c^{n-1}_{k-\frac{1}{2},\, \ell-\frac{1}{2}} \; .$$

The computational network sequence is chosen such that on each constant n plane k varies along constant ℓ strips.

On the nth plane, r and z are approximated using the velocity definitions of equation (4).

$$(11) \qquad \left\{ \begin{aligned} r^{n}_{k,\ell} &= r^{n-1}_{k,\ell} + u^{n-\frac{1}{2}}_{k,\ell}\, \Delta t^{n-\frac{1}{2}} \quad , \\ z^{n}_{k,\ell} &= z^{n-1}_{k,\ell} + v^{n-\frac{1}{2}}_{k,\ell}\, \Delta t^{n-\frac{1}{2}} \quad . \end{aligned} \right.$$

At half points r, z are obtained by averaging the values at the neighboring whole points.

$$
(12)\quad \left\{
\begin{aligned}
r^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= \tfrac{1}{4}(r^n_{k-1,\ell-1} + r^n_{k,\ell-1} + r^n_{k-1,\ell} + r^n_{k,\ell}) \quad , \\
z^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= \tfrac{1}{4}(z^n_{k-1,\ell-1} + z^n_{k,\ell-1} + z^n_{k-1,\ell} + z^n_{k,\ell}) \quad .
\end{aligned}
\right.
$$

From these values the gradients of r,z are approximated by the difference quotients

$$
(13)\quad \left\{
\begin{aligned}
\left(\frac{\partial r}{\partial R}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= \frac{1}{\Delta R_{k-\frac{1}{2},\ell-\frac{1}{2}}}\left(\frac{r^n_{k,\ell-1} + r^n_{k,\ell}}{2} - \frac{r^n_{k-1,\ell-1} + r^n_{k-1,\ell}}{2}\right) , \\
\left(\frac{\partial z}{\partial R}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= \frac{1}{\Delta R_{k-\frac{1}{2},\ell-\frac{1}{2}}}\left(\frac{z^n_{k,\ell-1} + z^n_{k,\ell}}{2} - \frac{z^n_{k-1,\ell-1} + z^n_{k-1,\ell}}{2}\right) , \\
\left(\frac{\partial r}{\partial Z}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= \frac{1}{\Delta Z_{k-\frac{1}{2},\ell-\frac{1}{2}}}\left(\frac{r^n_{k-1,\ell} + r^n_{k,\ell}}{2} - \frac{r^n_{k-1,\ell-1} + r^n_{k,\ell-1}}{2}\right) , \\
\left(\frac{\partial z}{\partial Z}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= \frac{1}{\Delta Z_{k-\frac{1}{2},\ell-\frac{1}{2}}}\left(\frac{z^n_{k-1,\ell} + z^n_{k,\ell}}{2} - \frac{z^n_{k-1,\ell-1} + z^n_{k,\ell-1}}{2}\right) ,
\end{aligned}
\right.
$$

which give for the Jacobian

$$(14) \qquad J^n_{k-\frac{1}{2},\ell-\frac{1}{2}} = \begin{vmatrix} \left(\frac{\partial r}{\partial R}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}} & \left(\frac{\partial r}{\partial Z}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}} \\ \left(\frac{\partial z}{\partial R}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}} & \left(\frac{\partial z}{\partial Z}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}} \end{vmatrix} .$$

The continuity equation (2) now gives the density as

$$(15) \qquad \rho_{k-\frac{1}{2},\ell-\frac{1}{2}} = \frac{w^o_{k-\frac{1}{2},\ell-\frac{1}{2}}}{r^n_{k-\frac{1}{2},\ell-\frac{1}{2}} \; J^n_{k-\frac{1}{2},\ell-\frac{1}{2}}} ,$$

which gives the approximate time derivative of the density as

$$(16) \qquad \left(\frac{\partial\rho}{\partial t}\right)^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}} = \frac{\rho^n_{k-\frac{1}{2},\ell-\frac{1}{2}} - \rho^{n-1}_{k-\frac{1}{2},\ell-\frac{1}{2}}}{(\Delta t)^{n-\frac{1}{2}}} .$$

The artificial viscosity term is evaluated from (7):

$$(17) \qquad q^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}} = \begin{cases} \dfrac{\alpha\left(\rho^o_{k-\frac{1}{2},\ell-\frac{1}{2}}\right)^2 \frac{1}{2}\left(J^n_{k-\frac{1}{2},\ell-\frac{1}{2}} + J^{n-1}_{k-\frac{1}{2},\ell-\frac{1}{2}}\right)}{\frac{1}{8}\left(\rho^n_{k-\frac{1}{2},\ell-\frac{1}{2}} + \rho^{n-1}_{k-\frac{1}{2},\ell-\frac{1}{2}}\right)^3} \cdots \\ \qquad \cdots \left[\left(\frac{\partial\rho}{\partial t}\right)^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}}\right]^2 \\ \qquad \text{if} \quad \left(\frac{\partial\rho}{\partial t}\right)^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}} > 0 , \\ 0 \quad \text{if} \quad \left(\frac{\partial\rho}{\partial t}\right)^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}} \le 0 . \end{cases}$$

The energy conservation equation (5) and the equation of state (8) can be used to calculate the thermodynamic variables $E^n_{k-\frac{1}{2},\ell-\frac{1}{2}}$, $p^n_{k-\frac{1}{2},\ell-\frac{1}{2}}$, and $c^n_{k-\frac{1}{2},\ell-\frac{1}{2}}$ by simultaneous solution:

$$
(18)\quad \left\{
\begin{aligned}
E^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= (\Delta t)^{n-\frac{1}{2}}\left[E^{n-1}_{k-\frac{1}{2},\ell-\frac{1}{2}} + \frac{1}{2}\left(\frac{p^n_{k-\frac{1}{2},\ell-\frac{1}{2}} + q^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}}}{(\rho^n_{k-\frac{1}{2},\ell-\frac{1}{2}})^2}\right.\right. \\
&\qquad \left.\left. + \frac{p^{n-1}_{k-\frac{1}{2},\ell-\frac{1}{2}}}{(\rho^{n-1}_{k-\frac{1}{2},\ell-\frac{1}{2}})^2}\right) \left(\frac{\partial\rho}{\partial t}\right)^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}}\right], \\
p^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= p\left(E^n_{k-\frac{1}{2},\ell-\frac{1}{2}},\ \rho^n_{k-\frac{1}{2},\ell-\frac{1}{2}}\right), \\
c^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= c\left(E^n_{k-\frac{1}{2},\ell-\frac{1}{2}},\ \rho^n_{k-\frac{1}{2},\ell-\frac{1}{2}}\right).
\end{aligned}
\right.
$$

The second and third equations in (18) will differ for air and explosion gas. P is approximated by

$$
P^n_{k-\frac{1}{2},\ell-\frac{1}{2}} = p^n_{k-\frac{1}{2},\ell-\frac{1}{2}} + q^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}} .
$$

With the quantities computed previously, the following can be calculated:

$$
(19)\quad \left\{
\begin{aligned}
A^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= \left[\left(\frac{\partial r}{\partial R}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}}\right]^2 + \left[\left(\frac{\partial z}{\partial R}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}}\right]^2 \\
&\quad + \left[\left(\frac{\partial r}{\partial Z}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}}\right]^2 + \left[\left(\frac{\partial z}{\partial Z}\right)^n_{k-\frac{1}{2},\ell-\frac{1}{2}}\right]^2 , \\
B^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= \left| \frac{J^n_{k-\frac{1}{2},\ell-\frac{1}{2}}}{c_{k-\frac{1}{2},\ell-\frac{1}{2}} \left(1.25\, A^n_{k-\frac{1}{2},\ell-\frac{1}{2}}\right)^{\frac{1}{2}}} \right| , \\
C^n_{k-\frac{1}{2},\ell-\frac{1}{2}} &= \begin{cases}
B^n_{k-\frac{1}{2},\ell-\frac{1}{2}} & \text{if} \quad \left(\frac{\partial \rho}{\partial t}\right)^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}} \leq 0 , \\
\left| \dfrac{0.225\, \rho^n_{k-\frac{1}{2},\ell-\frac{1}{2}}\, J^n_{k-\frac{1}{2},\ell-\frac{1}{2}}}{\alpha\, A^n_{k-\frac{1}{2},\ell-\frac{1}{2}} \left(\frac{\partial \rho}{\partial t}\right)^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}}} \right| & \\
\qquad \text{if} \quad \left(\frac{\partial \rho}{\partial t}\right)^{n-\frac{1}{2}}_{k-\frac{1}{2},\ell-\frac{1}{2}} > 0 . &
\end{cases}
\end{aligned}
\right.
$$

Minimizing, the following is obtained:

If $k = 1$, $\ell = 1$,

(20) $$D^n_{\frac{1}{2},\frac{1}{2}} = \mathrm{MIN}\left(B^n_{\frac{1}{2},\frac{1}{2}}, C^n_{\frac{1}{2},\frac{1}{2}}\right).$$

If $k = 1$, $\ell > 1$,

(21) $$D^n_{\frac{1}{2},\ell-\frac{1}{2}} = \mathrm{MIN}\left(B^n_{\frac{1}{2},\ell-\frac{1}{2}}, C^n_{\frac{1}{2},\ell-\frac{1}{2}}, D^n_{\max(k^*-\frac{1}{2}),\ell-1\frac{1}{2}}\right).$$

If $k > 1$,

(22) $$D^n_{k-\frac{1}{2},\ell-\frac{1}{2}} = \mathrm{MIN}\left(B^n_{k-\frac{1}{2},\ell-\frac{1}{2}}, C^n_{k-\frac{1}{2},\ell-\frac{1}{2}}, D^n_{k-1\frac{1}{2},\ell-\frac{1}{2}}\right).$$

(Note: The calculation above assumes that k increases on constant ℓ strips.)

Time increments are calculated from

(23) $$\begin{cases} \Delta t^{n+\frac{1}{2}} = \mathrm{MIN}\left(1.4\,\Delta t^{n-\frac{1}{2}}, D^n_{\max(k-\frac{1}{2}),\max(\ell-\frac{1}{2})}\right), \\ \Delta t^n = \frac{1}{2}\left(\Delta t^{n-\frac{1}{2}} + \Delta t^{n+\frac{1}{2}}\right), \end{cases}$$

and the time from

(24) $$\begin{cases} t^n = t^{n-1} + \Delta t^{n-\frac{1}{2}}, \\ t^{n+\frac{1}{2}} = t^{n-\frac{1}{2}} + \Delta t^n. \end{cases}$$

The P, r, z gradients at k-1,ℓ-1 are approximated by:

$$
(25)\quad \left\{
\begin{aligned}
\left(\frac{\partial P}{\partial R}\right)^{n}_{k-1,\ell-1} &= \frac{1}{\Delta R_{k-1,\ell-1}}\left(\frac{P^{n}_{k-\frac{1}{2},\ell-1\frac{1}{2}} + P^{n}_{k-\frac{1}{2},\ell-\frac{1}{2}}}{2} - \frac{P^{n}_{k-1\frac{1}{2},\ell-1\frac{1}{2}} + P^{n}_{k-1\frac{1}{2},\ell-\frac{1}{2}}}{2}\right), \\
\left(\frac{\partial P}{\partial Z}\right)^{n}_{k-1,\ell-1} &= \frac{1}{\Delta Z_{k-1,\ell-1}}\left(\frac{P^{n}_{k-1\frac{1}{2},\ell-\frac{1}{2}} + P^{n}_{k-\frac{1}{2},\ell-\frac{1}{2}}}{2} - \frac{P^{n}_{k-1\frac{1}{2},\ell-1\frac{1}{2}} + P^{n}_{k-\frac{1}{2},\ell-1\frac{1}{2}}}{2}\right), \\
\left(\frac{\partial r}{\partial R}\right)^{n}_{k-1,\ell-1} &= \frac{1}{\Delta R_{k-1,\ell-1}}\left(\frac{r^{n}_{k-\frac{1}{2},\ell-1\frac{1}{2}} + r^{n}_{k-\frac{1}{2},\ell-\frac{1}{2}}}{2} - \frac{r^{n}_{k-1\frac{1}{2},\ell-1\frac{1}{2}} + r^{n}_{k-1\frac{1}{2},\ell-\frac{1}{2}}}{2}\right), \\
\left(\frac{\partial r}{\partial Z}\right)^{n}_{k-1,\ell-1} &= \frac{1}{\Delta Z_{k-1,\ell-1}}\left(\frac{r^{n}_{k-1\frac{1}{2},\ell-\frac{1}{2}} + r^{n}_{k-\frac{1}{2},\ell-\frac{1}{2}}}{2} - \frac{r^{n}_{k-1\frac{1}{2},\ell-1\frac{1}{2}} + r^{n}_{k-\frac{1}{2},\ell-1\frac{1}{2}}}{2}\right), \\
\left(\frac{\partial z}{\partial R}\right)^{n}_{k-1,\ell-1} &= \frac{1}{\Delta R_{k-1,\ell-1}}\left(\frac{z^{n}_{k-\frac{1}{2},\ell-1\frac{1}{2}} + z^{n}_{k-\frac{1}{2},\ell-\frac{1}{2}}}{2} - \frac{z^{n}_{k-1\frac{1}{2},\ell-1\frac{1}{2}} + z^{n}_{k-1\frac{1}{2},\ell-\frac{1}{2}}}{2}\right), \\
\left(\frac{\partial z}{\partial Z}\right)^{n}_{k-1,\ell-1} &= \frac{1}{\Delta Z_{k-1,\ell-1}}\left(\frac{z^{n}_{k-1\frac{1}{2},\ell-\frac{1}{2}} + z^{n}_{k-\frac{1}{2},\ell-\frac{1}{2}}}{2} - \frac{z^{n}_{k-1\frac{1}{2},\ell-1\frac{1}{2}} + z^{n}_{k-\frac{1}{2},\ell-1\frac{1}{2}}}{2}\right).
\end{aligned}
\right.
$$

From these results, K and L are evaluated as:

$$
(26)\quad
\begin{cases}
K^{n}_{k-1,\ell-1} = \begin{vmatrix} \left(\dfrac{\partial P}{\partial R}\right)^{n}_{k-1,\ell-1} & \left(\dfrac{\partial z}{\partial R}\right)^{n} \\[2ex] \left(\dfrac{\partial P}{\partial Z}\right)^{n}_{k-1,\ell-1} & \left(\dfrac{\partial z}{\partial Z}\right)^{n}_{k-1,\ell-1} \end{vmatrix} , \\[6ex]
L^{n}_{k-1,\ell-1} = \begin{vmatrix} \left(\dfrac{\partial P}{\partial R}\right)^{n}_{k-1,\ell-1} & \left(\dfrac{\partial r}{\partial R}\right)^{n}_{k-1,\ell-1} \\[2ex] \left(\dfrac{\partial P}{\partial Z}\right)^{n}_{k-1,\ell-1} & \left(\dfrac{\partial r}{\partial Z}\right)^{n}_{k-1,\ell-1} \end{vmatrix} .
\end{cases}
$$

The momentum conservation equations (3) give the accelerations

$$
(27)\quad
\begin{cases}
\left(\dfrac{\partial u}{\partial t}\right)^{n}_{k-1,\ell-1} = - \dfrac{r^{n}_{k-1,\ell-1} \; K^{n}_{k-1,\ell-1}}{w^{o}_{k-1,\ell-1}} , \\[4ex]
\left(\dfrac{\partial v}{\partial t}\right)^{n}_{k-1,\ell-1} = \dfrac{r^{n}_{k-1,\ell-1} \; L^{n}_{k-1,\ell-1}}{w^{o}_{k-1,\ell-1}} ,
\end{cases}
$$

from which the velocities are derived:

$$
(28)\quad
\begin{cases}
u^{n+\frac{1}{2}}_{k-1,\ell-1} = u^{n-\frac{1}{2}}_{k-1,\ell-1} + \left(\dfrac{\partial u}{\partial t}\right)^{n}_{k-1,\ell-1} \Delta t^{n} , \\[4ex]
v^{n+\frac{1}{2}}_{k-1,\ell-1} = v^{n-\frac{1}{2}}_{k-1,\ell-1} + \left(\dfrac{\partial v}{\partial t}\right)^{n}_{k-1,\ell-1} \Delta t^{n} .
\end{cases}
$$

This completes the evaluation of all points away from the explosion gas-air boundaries.

Each network point k, ℓ is examined to determine if it crosses any boundary. Proceeding on successive constant Z or ℓ lines from R = 0 to R > 0 (Figure 4), starting from a negative $Z < (z_b^o - r_b^o)$ to some positive $Z > (z_a^o + r_a^o)$, the R = 0 point has entered the domain of the explosion gas b whenever

$$z_b^o - r_b^o \leq Z_{o,\ell} \leq z_b^o + r_b^o \quad ,$$

and has entered the domain of the explosion gas a whenever

$$z_a^o - r_a^o \leq Z_{o,\ell} \leq z_a^o + r_a^o \quad .$$

If the R = 0 point is inside the explosion gas b, along each $Z_{k,\ell} = Z_{o,\ell}$ = constant line, the inequality

$$R_{k,\ell}^2 - \left[(r_b^o)^2 - (Z_{o,\ell} - z_b^o)^2 \right] \leq 0 \quad ,$$

obtained using (9) must hold as $R_{k,\ell}$ increases. If the inequality holds, R is inside the explosion gas b; otherwise, R is in air. Similarly, if the R = 0 point is inside the explosion gas a, along each $Z_{k,\ell} = Z_{o,\ell}$ = constant line, if

$$R_{k,\ell}^2 - \left[(r_a^o)^2 - (Z_{o,\ell} - z_a^o)^2 \right] \leq 0 \quad .$$

is satisfied, R is inside a, and outside, if the inequality fails.

At any stage where R traverses an explosion gas boundary into air, J, K, L are evaluated by taking finite difference quotients across this discontinuity boundary. While this procedure is not exact, the error committed attenuates with propagation into the flow field.

CONCLUSION

The new BRLESC computing machine of BRL, with addition times in micro-seconds, can compute very rapidly the three dimensional network required in the numerical scheme of this problem. While the extensive information-hunting and reading with the magnetic tape will slow the overall computation considerably, a total of 52,000 fast core memory will be available soon, making the problem entirely feasible and the computing time very fast.

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